



# General Certificate of Education

## Mathematics 6360

### *MD02 Decision 2*

## Mark Scheme

### *2006 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

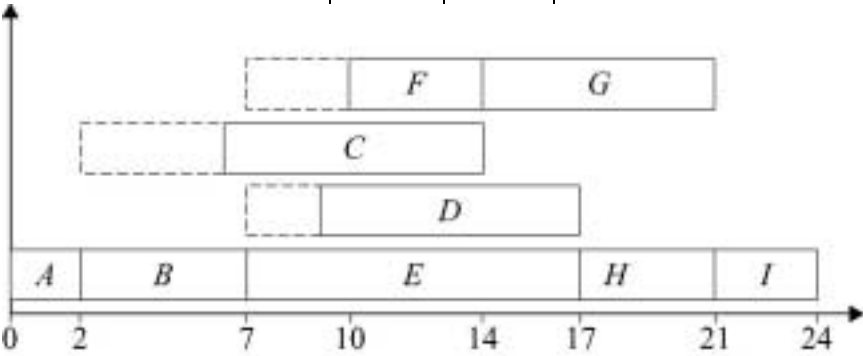
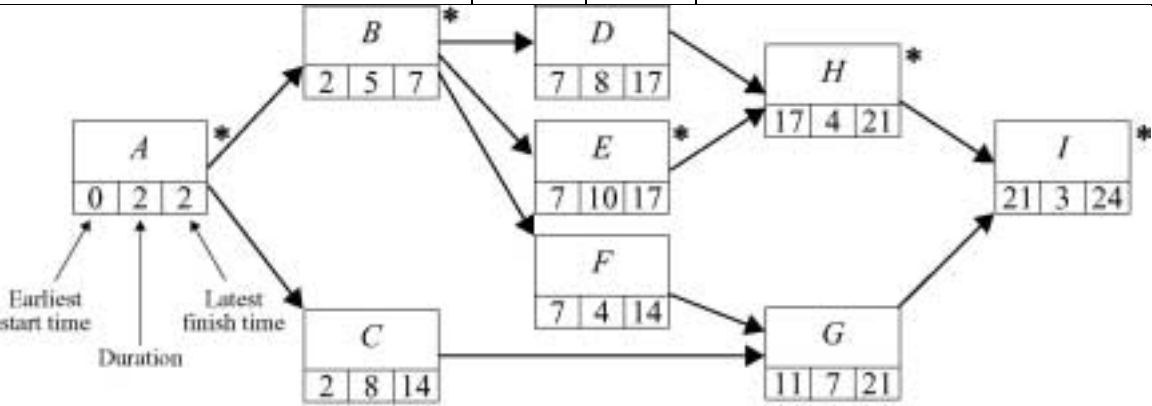
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

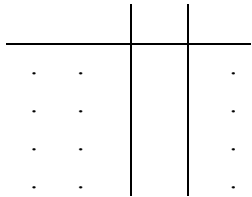
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

MD02

Q	Solution	Marks	Total	Comments																																																	
<p>1(a)</p> <p>(b) Forward pass for earliest start times</p> <p>(c) Backward pass for latest finish times</p> <p>(d) Critical path <i>A B E H I</i></p> <p>(e)</p> <table border="1" data-bbox="252 1189 660 1261"> <tr> <td>Non critical</td> <td><i>C</i></td> <td><i>D</i></td> <td><i>F</i></td> <td><i>G</i></td> </tr> <tr> <td>Float</td> <td>4</td> <td>2</td> <td>3</td> <td>3</td> </tr> </table> <p>(f)</p>  <p>'their' critical path on chart  C from 6 to 14 (with space 2-6)  D from 9 to 17 (with slack 7-9)  F &amp; G from 10 to 21 with appropriate slack</p>	Non critical	<i>C</i>	<i>D</i>	<i>F</i>	<i>G</i>	Float	4	2	3	3	 <table border="1" data-bbox="786 734 1013 1294"> <tr> <td>M1</td> <td></td> <td></td> <td>SCA</td> </tr> <tr> <td>A1</td> <td></td> <td></td> <td>(almost correct 2 slips)</td> </tr> <tr> <td>A1</td> <td>3</td> <td></td> <td>Correct</td> </tr> <tr> <td>M1</td> <td></td> <td></td> <td></td> </tr> <tr> <td>A1</td> <td>2</td> <td></td> <td>All correct</td> </tr> <tr> <td>M1</td> <td></td> <td></td> <td></td> </tr> <tr> <td>A1</td> <td>2</td> <td></td> <td>All correct</td> </tr> <tr> <td>B1</td> <td>1</td> <td></td> <td></td> </tr> <tr> <td>M1</td> <td></td> <td></td> <td>At least one float time correct</td> </tr> <tr> <td>A1</td> <td>2</td> <td></td> <td>All correct</td> </tr> </table>	M1			SCA	A1			(almost correct 2 slips)	A1	3		Correct	M1				A1	2		All correct	M1				A1	2		All correct	B1	1			M1			At least one float time correct	A1	2		All correct	<p>14</p>	<p>14</p>
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<b>Total</b>			<b>14</b>																																																		

**MD02 (cont)**

Q	Solution	Marks	Total	Comments																																				
<b>2(a)</b>	Add extra row with all values equal	B1	1	Usually + 25 and below rest 18    15    19    20    17 23    24    22    25    23 20    16    18    22    19 21    17    18    23    20 25    25    25    25    25																																				
<b>(b)</b>	Reduce columns first	M1		Do not award if full row of zeros added																																				
	<table style="margin-left: 40px;"> <tr><td></td><td>P</td><td>Q</td><td>R</td><td>S</td><td>T</td></tr> <tr><td>A</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>B</td><td>5</td><td>9</td><td>4</td><td>5</td><td>6</td></tr> <tr><td>C</td><td>2</td><td>1</td><td>0</td><td>2</td><td>2</td></tr> <tr><td>D</td><td>3</td><td>2</td><td>0</td><td>3</td><td>3</td></tr> <tr><td>(E)</td><td>7</td><td>10</td><td>7</td><td>5</td><td>8</td></tr> </table>		P	Q	R	S	T	A	0	0	1	0	0	B	5	9	4	5	6	C	2	1	0	2	2	D	3	2	0	3	3	(E)	7	10	7	5	8	A1		
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	Reduce rows next	M1		These 2 marks available for those who reduce row first																																				
	<table style="margin-left: 40px;"> <tr><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>5</td><td>0</td><td>1</td><td>2</td></tr> <tr><td>2</td><td>1</td><td>0</td><td>2</td><td>2</td></tr> <tr><td>3</td><td>2</td><td>0</td><td>3</td><td>3</td></tr> <tr><td>2</td><td>5</td><td>2</td><td>0</td><td>3</td></tr> </table>	0	0	1	0	0	1	5	0	1	2	2	1	0	2	2	3	2	0	3	3	2	5	2	0	3	A1✓													
0	0	1	0	0																																				
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2	1	0	2	2																																				
3	2	0	3	3																																				
2	5	2	0	3																																				
	Covering zeros requires 3 lines so adjust with least entry remaining being 1	M1		SC if full row of zeros, award M1 for further stage of adjustment and A1 for final correct matrix																																				
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	P	Q	R	S	T																																			
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C	1	0	0	2	1																																			
D	2	1	0	3	2																																			
E	1	4	2	0	2																																			
	Match: A-Tim; B-Phil; C-Quin; D-Ros	B1																																						
	Min <sup>m</sup> Time = 17 + 23 + 16 + 18 = 74 secs	B1	8																																					
	<b>Total</b>		<b>9</b>																																					

MD02 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Working back from H Starting from A (network)  $B \ 8^1$ $F \ 5^2 \ 4^3$ $C \ 7^1 \ 6^2$ $H \ 16^2 \ 14^4 \ 14^5$  $D \ 9^1 \ 6^2 \ 5^3$ $G \ 12^2 \ 8^4$ $E \ 8^1$	B1 M1 M1 M1  A1 A1	6	Alternatively, from A  First (stage) costs second stage attempt second stage indicated eg $15^2$ etc Third stage attempt (two numbers crossed out) Final value of 14 Dep on M2 earned All “correct” with 2 clear routes to cost of 14 (or equivalent in tabular form)
(b)	Min cost = 14 <i>ABCFH</i> and <i>ABCDGH</i>	B1 B1 B1	3	
<b>Total</b>			<b>9</b>	
4(a)	D	B1	1	
(b)	$(17 + 25 + 35 + 13 + 12 + 13 = 115)$	B1	1	
(c)	$ABD_{\max} = 25$ ; $GED_{\max} = 12$	B1B1	2	
(d)(i)	 Route <i>ABD</i> <i>GED</i> <i>GFD</i> <i>GD</i> <i>AD</i> <i>AFD</i> <i>GEED</i> Flow    25    12    16    13    17    15    7	M1 M1 M1 A1 A1  A1	6	Forward and backward flows Adjusting flows on diagram Routes and flows in chart One correct other than ABD, GED Another correct  All correct
(ii)	Total = 105 Max flow 	B1  B1	2	
(iii)	Cut through <i>AF</i> , <i>AD</i> , <i>BD</i> , <i>DE</i> , <i>DG</i> , and <i>GF</i>	M1 A1	2	Through 3 saturated arcs ( <i>fairly generous</i> ) Correct
(e)	Reduce max flow by their <i>EG</i> changing 19 to 15 $\Rightarrow$ New max = 101	M1 A1	2	Reduce by 4 since everywhere else saturated Correct answer $\Rightarrow$ 2 marks
<b>Total</b>			<b>16</b>	

**MD02 (cont)**

Q	Solution	Marks	Total	Comments																																			
<b>5(a)</b>	$3x+7y \leq 33$	M1	2	One correct inequality, or all using <																																			
	$x+2y \leq 10$ $2x+7y \leq 26$	A1		All correct																																			
<b>(b)(i)</b>	Compare $\frac{33}{3}, \frac{10}{1}, \frac{26}{2}$	E1	2																																				
	Choose smallest positive value $\Rightarrow$ pivot = 1	E1																																					
<b>(ii)</b>	<table border="1"> <thead> <tr> <th>P</th> <th>x</th> <th>y</th> <th>r</th> <th>s</th> <th>t</th> <th>Value</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>0</td> <td>-1</td> <td>0</td> <td>4</td> <td>0</td> <td>40</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>1</td> <td>-3</td> <td>0</td> <td>3</td> </tr> <tr> <td>0</td> <td>1</td> <td>2</td> <td>0</td> <td>1</td> <td>0</td> <td>10</td> </tr> <tr> <td>0</td> <td>0</td> <td><u>3</u></td> <td>0</td> <td>-2</td> <td>1</td> <td>6</td> </tr> </tbody> </table>	P	x	y	r	s	t	Value	1	0	-1	0	4	0	40	0	0	1	1	-3	0	3	0	1	2	0	1	0	10	0	0	<u>3</u>	0	-2	1	6	M1	7	Row operation
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		A1	Correct one row ( <i>other than pivot row</i> )																																				
	A1	All correct																																					
next y pivot on <u>3</u>	M1																																						
<table border="1"> <tbody> <tr> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td><math>3\frac{1}{3}</math></td> <td><math>\frac{1}{3}</math></td> <td>42</td> </tr> <tr> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td><math>-2\frac{1}{3}</math></td> <td><math>-\frac{1}{3}</math></td> <td>1</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td><math>2\frac{1}{3}</math></td> <td><math>-\frac{2}{3}</math></td> <td>6</td> </tr> <tr> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td><math>-\frac{2}{3}</math></td> <td><math>\frac{1}{3}</math></td> <td>2</td> </tr> </tbody> </table>	1	0	0	0	$3\frac{1}{3}$	$\frac{1}{3}$	42	0	0	0	1	$-2\frac{1}{3}$	$-\frac{1}{3}$	1	0	1	0	0	$2\frac{1}{3}$	$-\frac{2}{3}$	6	0	0	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	2	m1	Row operation									
1	0	0	0	$3\frac{1}{3}$	$\frac{1}{3}$	42																																	
0	0	0	1	$-2\frac{1}{3}$	$-\frac{1}{3}$	1																																	
0	1	0	0	$2\frac{1}{3}$	$-\frac{2}{3}$	6																																	
0	0	1	0	$-\frac{2}{3}$	$\frac{1}{3}$	2																																	
	A1	Correct one row (other than pivot row)																																					
	A1	All correct (condone multiples of given rows) (maximum 6 if y-pivot used first)																																					
<b>(iii)</b>	No negative number in top row	E1	3	ft if M3 scored and optimum reached																																			
	$P_{\max} = 42$	B1✓																																					
	$x = 6 \ y = 2$	B1✓																																					
<b>Total</b>			<b>14</b>																																				

**MD02 (cont)**

Q	Solution	Marks	Total	Comments
6(a)	Gain for Rowan + gain for Colleen in each strategy = 0	E1	1	Gain for one = loss of other
(b)	$  \begin{array}{ccc c}  & & & \underline{\text{min}} \\  -3 & -4 & 1 & -4 \\  1 & 5 & -1 & \boxed{-1} \\  -2 & -3 & 4 & -3 \\  \hline  \text{Max} & \boxed{1} & 5 & 4  \end{array}  $	M1 A1		$\left\{ \begin{array}{l} \text{minimum of rows \& max of columns} \\ \text{or} \\ \text{maximum of minima or minimax} \end{array} \right.$ All values correct (seen) or words maximin and minimax highlighted
	$1 \neq -1 \Rightarrow$ no stable solution	E1	3	
(c)	$R_3$ dominates $R_1$ $(-3, -4, 1) < (-2, -3, 4)$ so never play $R_1$	E1	1	
(d)(i)	R chooses $R_2$ with prob $p$ $\Rightarrow$ choose $R_3$ with prob $1 - p$ $\Rightarrow$ expected gain when C plays $C_1: p - 2(1 - p) = 3p - 2$ $C_2: 5p - 3(1 - p) = 8p - 3$ $C_3: -p + 4(1 - p) = 4 - 5p$ Plot expected gains for $0 \leq p \leq 1$	M1 A1 M1		Attempt at one expression
		A1		Condone mirror image
	Choosing their "highest" point $C_1$ & $C_3$ intersect $\Rightarrow 3p - 2 = 4 - 5p$ $\Rightarrow p = \frac{3}{4}$	M1 A1		Any 2 lines
	$\Rightarrow$ play $R_2$ with prob $\frac{3}{4}$ and $R_3$ with prob $\frac{1}{4}$	E1 $\checkmark$	7	Statement of strategy
(ii)	Value of game is $3 \times \frac{3}{4} - 2 = \frac{1}{4}$	B1	1	CSO or equivalent, eg 0.25
	<b>Total</b>		<b>13</b>	
	<b>TOTAL</b>		<b>75</b>	